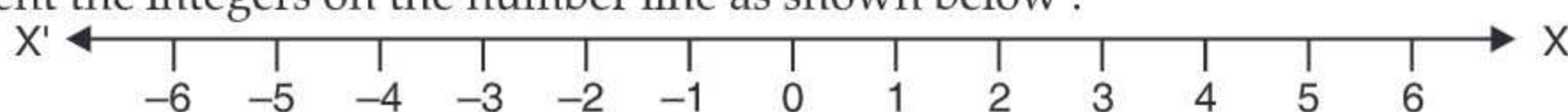


- **Natural Numbers** : Counting numbers starting from 1 are known as natural numbers and denoted by  $N$ .  
i.e.,  $N = \{1, 2, 3, 4, 5, \dots\}$
- **Whole Numbers** : All natural numbers together with 0 are called whole numbers and denoted by  $W$ .  
i.e.,  $W = \{0, 1, 2, 3, 4, 5, \dots\}$
- **Integers** : All natural numbers and negative of natural numbers including 0 are called integers.  
i.e.,  $\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots$  etc. are all integers  
We can represent the integers on the number line as shown below :



- **Rational Number** : A number is called rational if we can write the number in the form of  $\frac{p}{q}$ , where  $p$  and  $q$  are integers and  $q \neq 0$   
i.e.,  $1 = \frac{1}{1}, 2 = \frac{2}{1}, 0 = \frac{0}{1}$  and  $\frac{5}{8}, \frac{-3}{14}, \frac{7}{-15}$  are all rational numbers.
- Between two rational numbers  $x$  and  $y$ , there exists a rational number  $\frac{x+y}{2}$ .
- The idea of 'mean' helps us to find rational numbers between two given rational numbers.
- We can find countless rational numbers between two rational numbers.
- $-\frac{x}{y}$  is called the additive inverse of  $\frac{x}{y}$  and vice-versa.
- $\frac{y}{x}$  is called the multiplicative inverse or reciprocal of  $\frac{x}{y}$ .
- The rational number 0 is the additive identity for all rational numbers because a number does not change when 0 is added to it.
- The rational number 1 is the multiplicative identity for all rational numbers because on multiplying a rational number with 1, its value does not change.
- Rational numbers can be represented on a number line.
- The denominator of the rational number indicates the number of equal parts into which the first unit has been divided whereas the numerator indicates as to how many of these parts are to be taken into consideration.

➤ **Properties of Numbers :**

1. **Closure :**

(a) **Whole Numbers :**

Whole numbers are closed under :

- **Addition** : eg.  $0 + 8 = 8$  and  $5 + 6 = 11$  are whole numbers.
- **Multiplication** : eg.  $0 \times 5 = 0$  and  $4 \times 5 = 20$  are whole numbers.

Whole numbers are not closed under :

- **Subtraction** : eg.  $7 - 10 = -3$ , which is not a whole number.
- **Division** : eg.  $3 \div 7 = \frac{3}{7}$ , which is not a whole number.

(b) **Integers :**

Integers are closed under :

- **Addition** : eg.  $-7 + 3 = -4$ , an integer.
- **Subtraction** : eg.  $8 - 3 = 5$  and  $3 - 5 = -2$  are integers.
- **Multiplication** : eg.  $7 \times 9 = 63$  and  $-7 \times 9 = -63$  are integers.

Integers are not closed under :

- **Division** : eg.  $5 \div 9 = \frac{5}{9}$ , which is not an integer.

(c) **Rational Numbers :**

Rational numbers are closed under :

- **Addition :** eg.  $\frac{3}{5} + \frac{(-4)}{9} = \frac{27-20}{45} = \frac{7}{45}$  which is a rational number.

- **Subtraction :** eg.  $\frac{5}{8} - \frac{3}{7} = \frac{35-24}{56} = \frac{11}{56}$

and  $\frac{2}{5} - \frac{3}{4} = \frac{8-15}{20} = \frac{-7}{20}$

are rational numbers.

- **Multiplication :**

eg.  $\frac{-5}{7} \times \frac{2}{9} = \frac{-10}{63}$

and  $\frac{2}{3} \times \frac{5}{11} = \frac{10}{33}$

are rational numbers.

- **Division :** eg.  $\frac{-3}{5} \div \frac{2}{3} = \frac{9}{10}$ , which is also a rational number. For any rational number  $a$ ,  $a \div 0$  is not

defined. So, rational number are not closed under division.

However, if we exclude zero then the rational numbers are closed under division.

**2. Commutativity :**

(a) **Whole Numbers :**

Whole number are commutative under :

- **Addition :** eg.  $0 + 5 = 5 + 0 = 5$ , which is a whole number.
- **Multiplication :** eg.  $3 \times 5 = 5 \times 3 = 15$ , which is a whole number.

Whole numbers are not commutative under :

- **Subtraction :** eg.  $7 - 10 \neq 10 - 7$ .
- **Division :** eg.  $3 \div 5 \neq 5 \div 3 \Rightarrow \frac{3}{5} \neq \frac{5}{3}$ .

(b) **Integers :**

Integers are commutative under :

- **Addition :** eg.  $-5 + 7 = 7 - 5 = 2$ .
- **Multiplication :** eg.  $(-11) \times 3 = 3 \times (-11) = -33$ .

Integers are not commutative under :

- **Subtraction :** eg.  $10 - 13 \neq 13 - 10$ .
- **Division :** eg.  $-5 \div 8 \neq 8 \div (-5) \Rightarrow \frac{-5}{8} \neq \frac{8}{-5}$ .

(c) **Rational Numbers :**

- **Addition :** Two rational numbers can be added in any order, i.e., commutativity holds for rational numbers under addition, i.e., for any two rational number  $a$  and  $b$ ,  $a + b = b + a$ .

eg.  $\frac{-3}{4} + \frac{5}{11} = \frac{-13}{44}$

and  $\frac{5}{11} + \left(\frac{-3}{4}\right) = -\frac{13}{44}$

- **Subtraction :** eg.  $\frac{2}{5} - \frac{5}{6} = \frac{12-25}{30} = \frac{-13}{30}$

and  $\frac{5}{6} - \frac{2}{5} = \frac{25-12}{30} = \frac{13}{30}$

Hence, subtraction is not commutative for rational numbers.

**3. Associativity :**

(a) **Whole Numbers :**

Whole number are associative under :

- (a) Addition eg.  $6 + (5 + 4) = (6 + 5) + 4 = 15$       (b) Multiplication eg.  $2 \times (3 \times 6) = (2 \times 3) \times 6 = 36$ .

Whole number are not associative under :

- (a) Subtraction eg.  $7 - (2 - 1) = 7 - 1 = 6$  &  $(7 - 2) - 1 = 5 - 1 = 4$

- (b) Division eg.  $(8 \div 4) \div 2 = 2 \div 2 = 1$  &  $8 \div (4 \div 2) = 8 \div 2 = 4$

**(b) Integers :**

Integers are associative under :

(a) Addition eg.  $7 + (-5 + 2) = 7 + (-3) = 7 - 3 = 4$   
 $(7 - 5) + 2 = 2 + 2 = 4.$

(b) Multiplication eg.  $2 \times (3 \times 4) = 2 \times 12 = 24$   
 $(2 \times 3) \times 4 = 6 \times 4 = 24.$

Integers are not associative under :

(a) Subtraction :  $-7 - (5 - 2) = -7 - 3 = -10$   
 $(-7 - 5) - 2 = -12 - 2 = -14.$

(b) Division :  $(8 \div 2) \div (-2) = 4 \div (-2) = -2$   
 $8 \div [2 \div (-2)] = 8 \div (-1) = -8$

**(c) Rational Numbers :** Rational numbers are associative under.

(i) **Addition :** eg.  $\frac{-2}{5} + \left[ \frac{3}{4} + \left( \frac{-7}{8} \right) \right] = \frac{-2}{5} + \left( \frac{-1}{8} \right) = \frac{-21}{40} = \left[ \frac{-2}{5} + \frac{3}{4} \right] + \left( \frac{-7}{8} \right) = \frac{7}{20} - \frac{7}{8} = \frac{-21}{40}$

So, addition is associative for rational numbers, *i.e.*, for any three rational numbers  $a, b$  and  $c$ ,  
 $a + (b + c) = (a + b) + c.$

(ii) **Subtraction :** eg.  $\frac{-3}{4} - \left[ \frac{-5}{6} - \frac{2}{3} \right] = \frac{-3}{4} - \left( \frac{-9}{6} \right) = \left( \frac{-9}{6} \right) = \frac{9}{12} = \frac{3}{4}$

and  $\left[ \frac{-3}{4} - \left( \frac{-5}{6} \right) \right] - \frac{2}{3} = \frac{1}{12} - \frac{2}{3} = \frac{-7}{12}$

*i.e.*,  $\frac{3}{4} \neq \frac{-7}{12}.$

Hence, subtraction is not associative for rational numbers.

(iii) **Multiplication :**

eg.  $\frac{-2}{3} \times \left( \frac{2}{5} \times \frac{6}{7} \right) = \frac{-2}{3} \times \frac{12}{35} = \frac{-24}{105} = \frac{-8}{35}$

and  $\left( \frac{-2}{3} \times \frac{2}{5} \right) \times \frac{6}{7} = \frac{-4}{15} \times \frac{6}{7} = \frac{-24}{105} = \frac{-8}{35}$

So, multiplication is associative for rational number, *i.e.*, for any three rational numbers  $a, b$  and  $c$ ,  
 $a \times (b \times c) = (a \times b) \times c.$

(iv) **Division :**

eg.  $\frac{2}{3} \div \left[ \frac{-1}{2} \div \frac{2}{5} \right] = \left[ \frac{2}{3} \div \left( \frac{-1}{2} \right) \right] \div \frac{2}{5}$

We have, LHS  $\frac{2}{3} \div \left[ \frac{-1}{2} \div \frac{2}{5} \right] = \left[ \frac{2}{3} \div \frac{1}{2} \times \frac{5}{2} \right]$   
 $= \frac{2}{3} \div \left( \frac{-5}{4} \right) = \frac{2}{3} \times \frac{4}{-5} = \frac{8}{-15}$

RHS  $\left[ \frac{2}{3} \div \left( \frac{-1}{2} \right) \right] \div \frac{2}{5} = \left[ \frac{2}{3} \times \left( \frac{-2}{1} \right) \right] \div \frac{2}{5} = \left( \frac{-4}{3} \right) \div \frac{2}{5}$   
 $= \frac{-4}{3} \times \frac{5}{2} = \frac{-20}{6} = \frac{-10}{3}$

But LHS  $\neq$  RHS

Hence, division is not associative for rational numbers.

**4. Distributivity of multiplication over addition for rational number :** For all rational numbers  $a, b$  and  $c$ ,

$$a(b + c) = ab + ac$$

eg. Let  $\frac{-2}{5}, \frac{3}{8}$  and  $\frac{-3}{4}$  are any three rational numbers, then

$$\begin{aligned}\frac{-2}{5} \times \left\{ \frac{3}{8} + \left( \frac{-3}{4} \right) \right\} &= \frac{-2}{5} \times \left\{ \frac{3 + (-6)}{8} \right\} \\ &= \frac{-2}{5} \times \left( \frac{-3}{8} \right) = \frac{6}{40} = \frac{3}{20}\end{aligned}$$

Also, 
$$\frac{-2}{5} \times \frac{3}{8} = \frac{-6}{40} = \frac{-3}{20}$$

and 
$$\frac{-2}{5} \times \frac{-3}{4} = \frac{6}{20}$$

Therefore, 
$$\left( \frac{-2}{5} \times \frac{3}{8} \right) + \left( \frac{-2}{5} \times \frac{-3}{4} \right) = \frac{-3}{20} + \frac{6}{20} = \frac{3}{20}$$

Thus, 
$$\frac{2}{5} \times \left\{ \frac{3}{8} + \left( \frac{-3}{4} \right) \right\} = \left( \frac{-2}{5} \times \frac{3}{8} \right) + \left\{ \frac{-2}{5} \times \left( \frac{-3}{4} \right) \right\}$$

**5. Distributivity of multiplication over subtraction for rational number :** For any three rational numbers

$a, b$  and  $c, a(b - c) = ab - ac$

eg. Let  $\frac{1}{2}, \frac{-2}{5}$  and  $\frac{-3}{10}$  are any three rational numbers, then

$$\begin{aligned}\frac{1}{2} \times \left\{ \frac{-2}{5} - \left( \frac{-3}{10} \right) \right\} &= \frac{1}{2} \times \left\{ \frac{-4 - (-3)}{10} \right\} = \frac{1}{2} \times \left( \frac{-4 + 3}{10} \right) \\ &= \frac{1}{2} \times \left( \frac{-1}{10} \right) = \frac{-1}{20}\end{aligned}$$

Also, 
$$\frac{1}{2} \times \left( \frac{-2}{5} \right) = \frac{-2}{10} = -\frac{1}{5}$$

and 
$$\frac{1}{2} \times \left( \frac{-3}{10} \right) = \frac{-3}{20}$$

Therefore, 
$$\left( \frac{1}{2} \times \frac{-2}{5} \right) - \left( \frac{1}{2} \times \frac{-3}{10} \right) = \frac{-1}{5} - \left( \frac{-3}{20} \right)$$

$$= \frac{-1}{5} + \frac{3}{20}$$

$$= \frac{-4 + 3}{20} = \frac{-1}{20}$$

Thus, 
$$\frac{1}{2} \times \left\{ \frac{-2}{5} - \left( \frac{-3}{10} \right) \right\} = \left( \frac{1}{2} \times \frac{-2}{5} \right) - \left( \frac{1}{2} \times \frac{-3}{10} \right)$$

- **Equation** : An equation is a statement of equality which contains one or more unknown quantity/quantities (or variable/variables) is called an equation.
- **Linear equation in one variable** : An equation is called linear equation if it has only one degree *i.e.*, the highest power of the variable appearing in equation is 1, and the form of linear equation is

$$P(x) = ax + b = 0, \text{ e.g., } x + 5 = 0, \frac{x}{2} - 7 = 15.$$

- **Solving an equation** : Solving an equation means determining its root *i.e.*, determining (finding) the value of the variable which satisfies it.
- A linear equation may have any rational number, as its solution.
- An equation may have linear expression on both sides of the equation.
- Some equations may not be linear in the beginning, but they can be brought to be linear by using usual methods.
- The utility of linear equations is in their diverse applications, different problems on numbers, ages, perimeters, combination of currency notes and so on can be solved using linear equations.
- A number which satisfies an equation is called the solution of the equation.
- A term may be transposed from one side of the equation to the other side, but its sign will be changed.

**Ex. 1. Solve :**  $\frac{15}{4} - 7x = 9$

**Sol.** We have

$$\frac{15}{4} - 7x = 9$$

Transposing  $\frac{15}{4}$  to RHS

$$\begin{aligned} -7x &= 9 - \frac{15}{4} \\ -7x &= \frac{36 - 15}{4} = \frac{21}{4} \\ x &= \frac{21}{4 \times (-7)} \\ x &= \frac{-3}{4} \end{aligned}$$

Check

$$\begin{aligned} \text{LHS} &= \frac{15}{4} - 7 \times \left(\frac{-3}{4}\right) \\ &= \frac{15}{4} + \frac{21}{4} = \frac{36}{4} = 9 = \text{RHS} \end{aligned}$$

**Ex. 2. Solve :**  $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

**Sol.** Since LCM of 3 and 6 is 6. So multiply both sides by 6, we have

$$\frac{6(6x+1)}{3} + 6 = \frac{6(x-3)}{6}$$

$$2(6x+1) + 6 = x-3$$

$$12x + 2 + 6 = x - 3$$

$$12x - x = -3 - 2 - 6$$

$$11x = -11$$

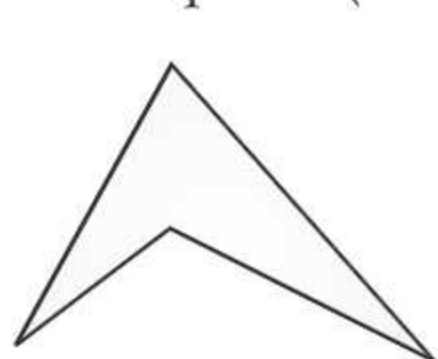
$$x = -1$$

$$\text{LHS} = \frac{6(-1)+1}{3} + 1 = \frac{-5}{3} + 1 = \frac{-2}{3}$$

$$= \frac{-1-3}{6} = \frac{-4}{6} = \frac{-2}{3}$$

$$\text{LHS} = \text{RHS}$$

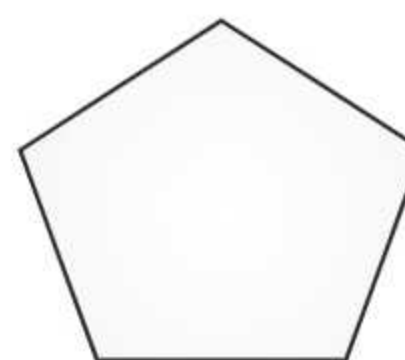
- **Curve** : A figure formed on a plane surface by joining a number of points without lifting a pencil is called a curve.
- **Open Curve** : A curve which does not end at the same starting point or which does not cut itself is called an open curve.
- **Closed Curve** : A curve which cut itself or which starts and ends at the same point is called closed curve.
- **Simple Closed Curve** : A closed curve called simple closed curve which does not intersect itself.
- **Polygon** : A polygon is a closed figure bounded by three or more line segments such that each line segment intersects exactly two other points (vertices) as shown in the following figures.



(i)



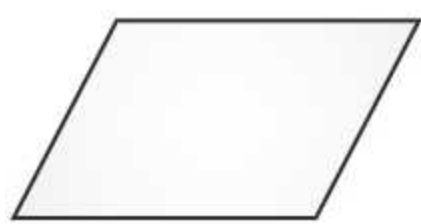
(ii)



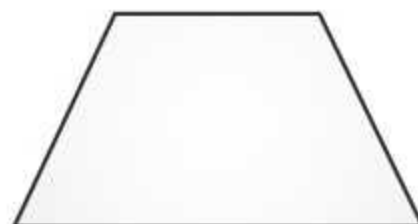
(iii)

- Polygons are classified into two types on the basis of interior angles :

- (a) **Convex Polygon** : In this case, each angle is either acute or obtuse ( $\text{angle} < 180^\circ$ ) as shown in the following figures.



(i)

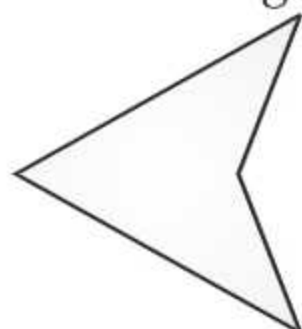


(ii)

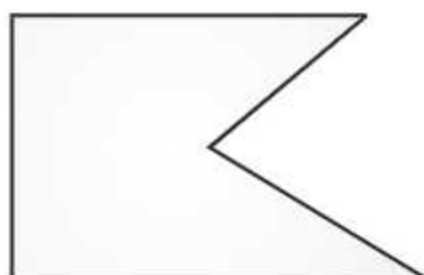


(iii)

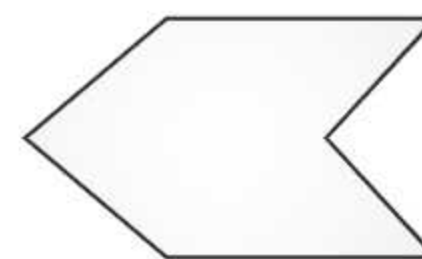
- (b) **Concave Polygon** : In this case, any one angle is reflex ( $\text{angle} > 180^\circ$ ) and one diagonal is outside the polygon as shown in the following figures.



(i)



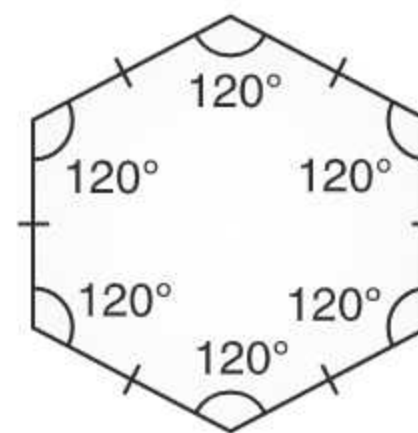
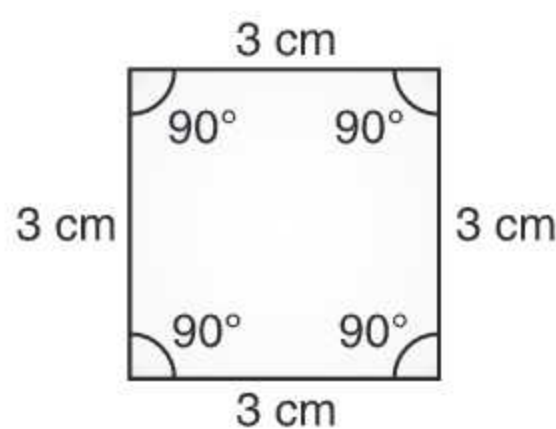
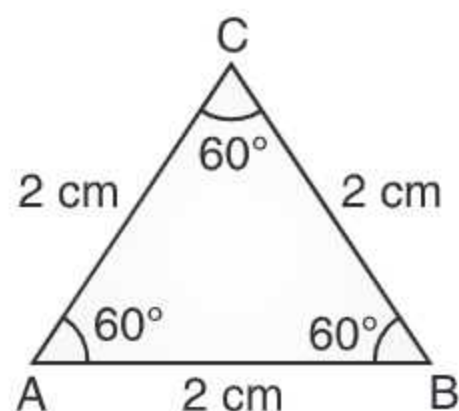
(ii)



(iii)

- On the basis of sides, there are two types of polygons :

- (a) **Regular Polygon** : A convex polygon is called a regular polygon, if all its sides and angles are equal as shown in the following figures.



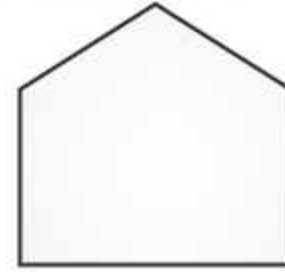
Each angle of a regular polygon of  $n$ -sides =  $\left(\frac{n-2}{n}\right) \times 180^\circ$ .

### Important results on polygon :

For a regular polygon of  $n$  sides ( $n \geq 3$ )

- Each interior angle =  $\left(\frac{n-2}{n} \times 180^\circ\right)$
- Each exterior angle =  $\left(\frac{360^\circ}{n}\right)$
- Interior angle + Exterior angle =  $180^\circ$   
For a convex polygon of  $n$  sides
- Sum of all interior angle =  $(n-2) \times 180^\circ$
- Sum of all exterior angles =  $360^\circ$
- Number of diagonals of  $n$  sides ( $n > 3$ ) =  $\frac{n(n-3)}{2}$

(b) **Irregular Polygon** : A polygon in which all the sides are unequal as shown in the following figures,

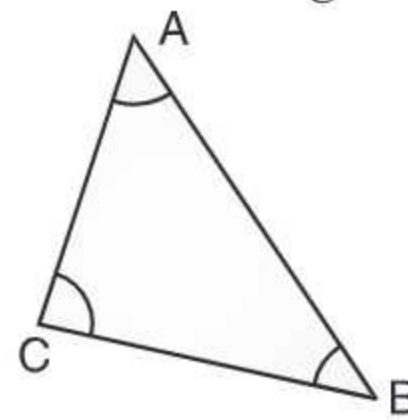


(i)



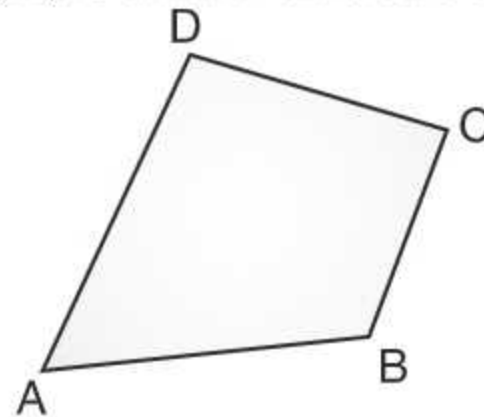
(ii)

➤ **Triangle** : A simple closed figure bounded by three line segments is called a triangle, it has three sides as  $AB$ ,  $BC$  and  $AC$ ; three vertices as  $A$ ,  $B$  and  $C$  and three interior angles  $\angle A$ ,  $\angle B$  and  $\angle C$  and the sum of all angles is  $180^\circ$ .



*i.e.*,  $\angle A + \angle B + \angle C = 180^\circ$

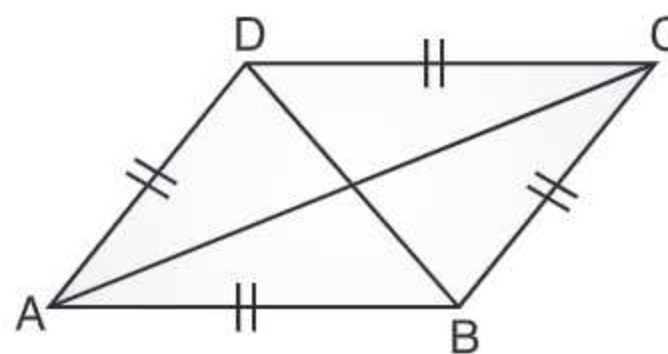
➤ **Quadrilateral** : A simple closed figure bounded by four line segments is called a quadrilateral, it has four sides *i.e.*,  $AB$ ,  $BC$ ,  $CD$  and  $AD$  and four vertices as  $A$ ,  $B$ ,  $C$  and  $D$  and the sum of all angles of a quadrilateral is  $360^\circ$ .



(a) **Parallelogram** : A quadrilateral in which opposite sides are parallel and equal is called parallelogram; written as  $\parallel gm.$  and the diagonals of a parallelogram bisect each other.

**Properties :**

- Opposite sides are equal.
- Opposite angles are equal.
- Diagonals bisect each other.



(b) **Rectangle** : A parallelogram each of whose angle is  $90^\circ$  and diagonals are equal, is called a rectangle.

**Properties :**

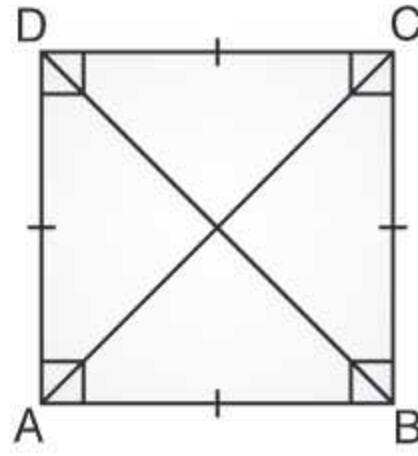
- Opposite sides are equal.
- Each angle is a right angle.
- Diagonals are equal.
- Diagonals bisect each other.



(c) **Square** : A quadrilateral in which all sides and angles are equal, is called a square.

**Properties :**

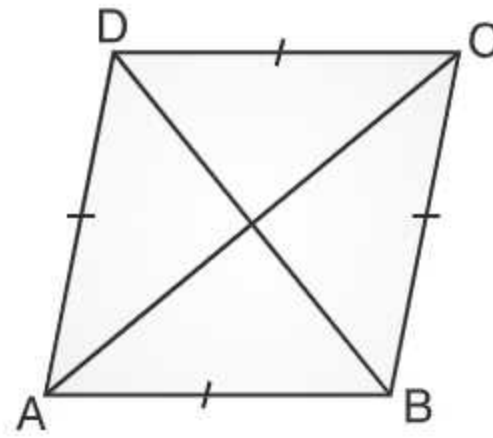
- All the sides are equal.
- Each angle is a right angle.
- Diagonals are equal.
- Diagonals bisect each other at right angle.



(d) **Rhombus** : A parallelogram having all its sides equal, is called a rhombus.

**Properties :**

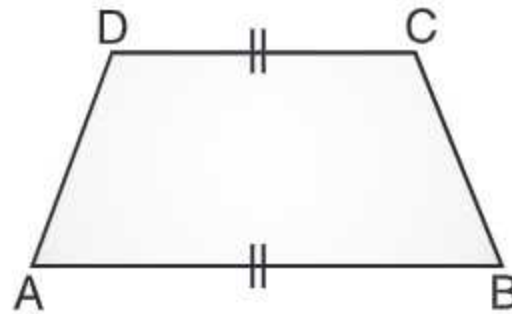
- All the side are equal.
- Opposite angles are equal.
- Diagonals bisect each other at right angle.



(e) **Trapezium** : A quadrilateral in which two opposite sides are parallel and other two opposite sides are non-parallel, is called a trapezium.

- If two non-parallel sides of a trapezium are equal, then it is called an isosceles trapezium.

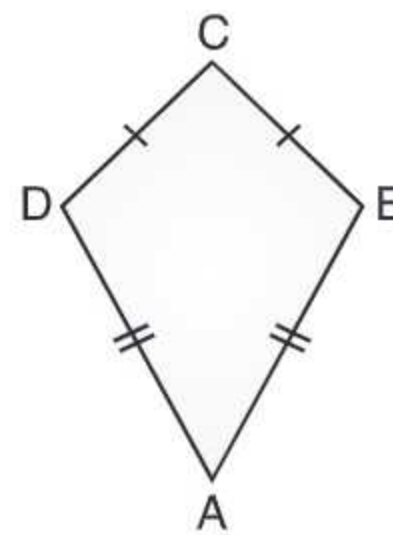
The line segment joining the mid-points of non-parallel sides of a trapezium is called its median.



(f) **Kite** : A quadrilateral in which two pairs of adjacent sides are equal, is called a kite.

**Properties :**

- Diagonals bisect each other at right angle.
- In the figure,  $m \angle B = m \angle D$ , but  $m \angle A \neq m \angle C$

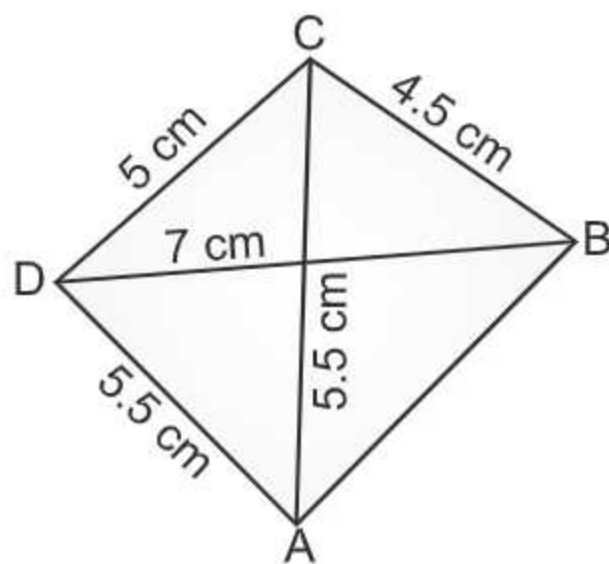


## Chapter 4 – Practical Geometry

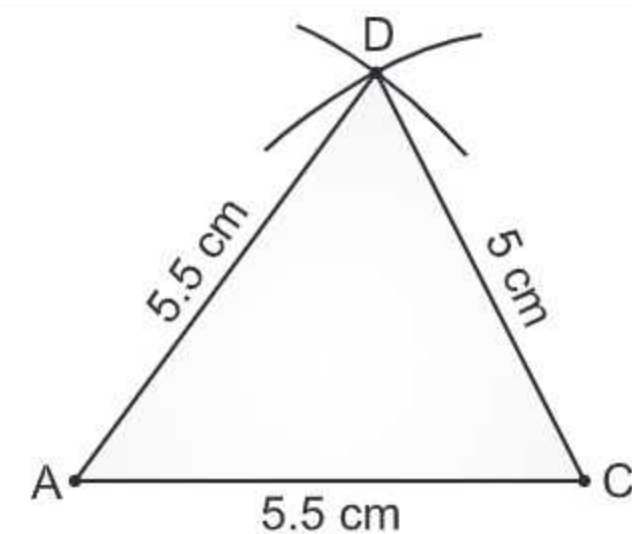
- To construct a quadrilateral uniquely, it is necessary to have the knowledge of at least five of its parts.
- Five necessary parts to construct a quadrilateral may be :
  - (A) four sides and diagonal
  - (B) Three sides and two diagonals
  - (C) Four sides and an angle
  - (D) Three sides and two included angles
  - (E) Two adjacent sides and three angles
- To construct a rhombus :
  - (A) Draw a diagonal of given length and draw perpendicular bisector of this diagonal base.
  - (B) Take half of the given measurement of second diagonal and cut off arcs on either side of the perpendicular bisector. It will give two points of rhombus.
  - (C) Join these points with the points on the first diagonal. It will give the required rhombus.
- Before constructing a quadrilateral, one must draw a rough sketch of given measurements.
- Trapezium, rhombus and square are different forms of a parallelogram.
- A rhombus can be a parallelogram, but a parallelogram cannot be a rhombus.
- A square and a rhombus both have equal sides and a square can be rhombus but a rhombus cannot be a square.

**Ex.** Construct a quadrilateral ABCD, give that BC = 4.5 cm, AD = 5.5 cm, CD = 5 cm the diagonal AC = 5.5 cm diagonal BD = 7 cm.

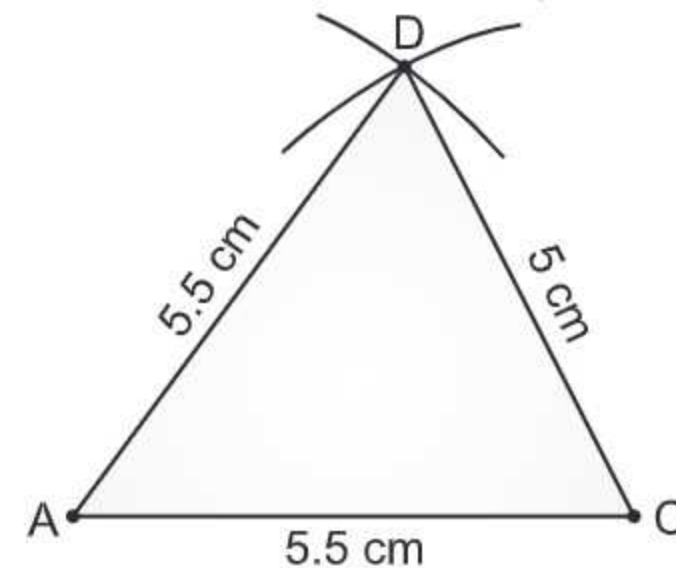
**Sol.** Here is the rough sketch of the quadrilateral ABCD (Fig 4.10). Studying this sketch, we can easily see that it is possible to draw  $\triangle ACD$  first (How ?).



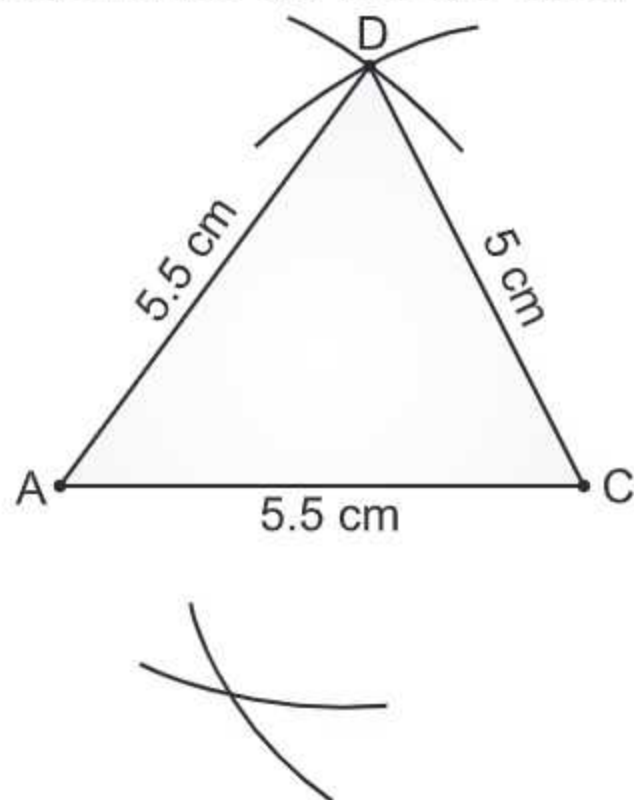
**Step I :** Draw  $\triangle ACD$  using SSS construction (Fig 4.11).  
(We now need to find B at a distance of 4.5 cm from C and 7 cm from D).



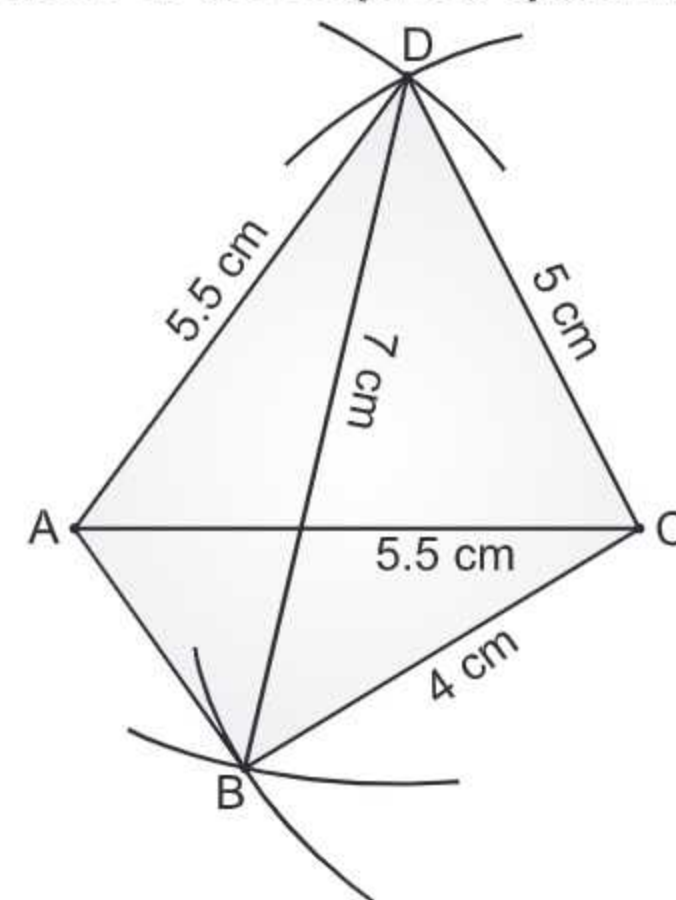
**Step II :** With D as centre, draw an arc of radius 7 cm. (B is somewhere on this arc) (Fig 4.12).



**Step III :** With C as centre, draw an arc of radius 4.5 cm (B is somewhere on this arc also) (Fig 4.13).



**Step IV :** Since B lies on both the arcs, B is the point intersection of the two arcs. Mark B and complete ABCD. ABCD is the required quadrilateral.



- **Data** : Numerical observations collected by an observer is called a data (raw data).
- **Frequency** : The number of times an observation occurs in the given data is called the frequency of the observation.
- **Frequency Distribution** : A way of presenting data that exhibits the values of the variable and corresponding frequencies is called a frequency distribution.
- **Range** : The difference between the highest and the lowest values of observation is called range.

$$\text{i.e.,} \quad \text{Range} = \text{Highest observation} - \text{Lowest observation}$$

- **Class-mark** : The mid-point of a class-interval is called class-mark.

$$\text{Class-mark} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

- **Histogram** : Grouped data can be presented using histogram. Histogram is a type of bar diagram, where the class intervals are shown on the horizontal axis and the height of the bars show the frequency of the class interval. Also, there is no gap between the bars as there is no gap between the class intervals.
- **Pie-chart or Pie-diagram** : A pie-chart is a pictorial representation of the numerical data by non-intersecting adjacent sectors of the circle such that area of each sector is proportional to the magnitude of the data represented by the sector.

$$\text{i.e.,} \quad \text{Central angle} = \frac{\text{Value of a particular mode}}{\text{Sum of the value}} \times 360^\circ$$

- **Probability** : Probability is the possibility and impossibility of an event ( $E$ ).

$$\text{i.e.,} \quad \text{Probability of an event } (E) = \frac{\text{Favourable outcomes}}{\text{Total number of possible outcomes}}$$

and

$$P(E) + P(\text{not } E) = 1, \text{ i.e., } 0 \leq P(E) \leq 1$$

- Probability of an event can have a value from 0 to 1.
- Probability to a sure event is 1.
- Probability of an impossible event is 0.
- If tossing a coin  $n$  times, then

$$\text{Total number of outcomes} = 2^n \quad [ \because \text{A coin has two faces} ]$$

e.g., A coin is tossed two times, then

$$\text{Total number of outcomes} = 2^n = 2^2 = 4$$

i.e.,

$$S = \{(HH), (HT), (TH), (TT)\}$$

- If rolling a die  $n$ -times, then

$$\text{Total number of outcomes} = 6^n \quad [ \because \text{A die has six faces} ]$$

e.g., A die is throwing once, then

$$\text{Total number of outcomes} = 6^n = 6^1 = 6$$

- **Random Experiment** : A random experiment is one whose outcome cannot be predicted exactly in advance.

**Ex.** On a particular day, the sales (in rupees) of different items of a baker's shop are given as follows.

Ordinary bread	:	320
Fruit bread	:	80
Cakes and pastries	:	160
Biscuits	:	120
Others	:	40
<b>Total</b>	:	<b>720</b>

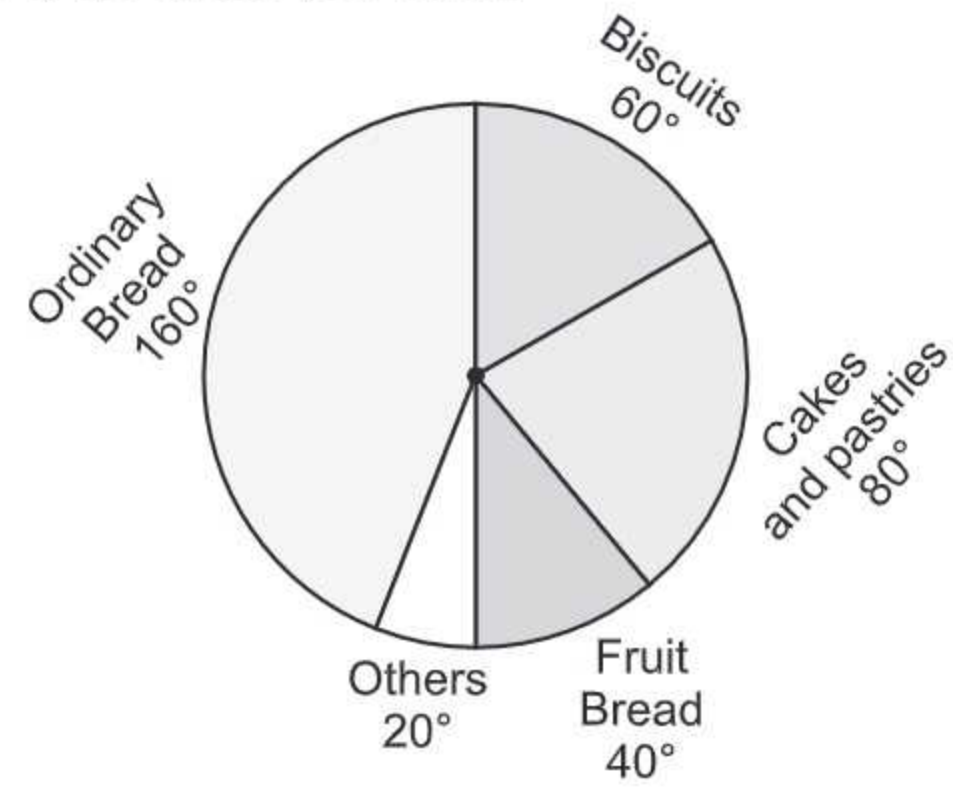
**Sol. Step I :** We find the central angle of each Sector. Here the total sale = ₹ 720. For this we make the table.

Item	Sales (in ₹)	In Fraction	Central Angle
Ordinary Bread	320	$\frac{320}{720} = \frac{4}{9}$	$\frac{4}{9} \times 360^\circ = 160^\circ$
Biscuits	120	$\frac{120}{720} = \frac{1}{6}$	$\frac{1}{6} \times 360^\circ = 60^\circ$
Cakes and pastries	160	$\frac{160}{720} = \frac{2}{9}$	$\frac{2}{9} \times 360^\circ = 80^\circ$

Fruit Bread	80	$\frac{80}{720} = \frac{1}{9}$	$\frac{1}{9} \times 360^\circ = 40^\circ$
Others	40	$\frac{40}{720} = \frac{1}{18}$	$\frac{1}{18} \times 360^\circ = 20^\circ$

**Step II :** Draw a circle for any radius and using protractor mark different angles according to above table.

Now, we make pie chart



**Square Number :**

- The square of a number is the product of the number with the number itself. Thus, square of  $x = (x \times x)$ , denoted by  $x^2$ .
- A natural number  $n$  is a perfect square, if  $n = m^2$  for some natural number  $m$ .  $\{1 = 1 \times 1 = 1^2, 4 = 2 \times 2 = 2^2\}$

**Square root :**

- Square root is the inverse operation of square, *i.e.*, positive square root of a number is denoted by the symbol  $\sqrt{\quad}$

For example,  $3^2 = 9$  gives  $\sqrt{9} = 3$  or  $(3^2)^{\frac{1}{2}} = 3$ .

- For positive numbers  $a$  and  $b$ , we have

$$(i) \sqrt{ab} = \sqrt{a} \times \sqrt{b}, \quad (ii) \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ or } \sqrt{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{2}}$$

**Properties of the Square Number :**

- A number ending in 2, 3, 7 or 8 is never a perfect square.
- A number ending in an odd number of zeros is never a perfect square.
- The square of an even number is even.
- The square of an odd number is odd.
- The square of a proper fraction is smaller than the fraction.
- For every natural number  $n$ , we have  $\{(n + 1)^2 - n^2\} = \{(n + 1) + n\}$ .
- Sum of first  $n$  odd natural numbers  $= n^2$ .
- If  $m, n, p$  are natural numbers such that  $(m^2 + n^2) = p^2$ , then  $(m, n, p)$  is called a Pythagorean triplet.
- For every natural number  $m > 1$ ,  $(2m, m^2 - 1, m^2 + 1)$  is a Pythagorean triplet.
- There are  $2n$  non perfect square numbers between the squares of the number  $n$  and  $(n + 1)$ .

**Ex.** Find the square root of 4096.

**Sol. Step I :** Place a bar over every pair of digits starting from the one's digit.  $(\overline{4096})$ .

**Step II :** Find the largest number whose square is less than or equal to the number under the left-most bar ( $62 < 40 < 72$ ). Take this number as the divisor and the number under the left-most bar as the dividend. Divide and get the remainder *i.e.*, 4 in this case.

**Step III :** Bring down the number under the next bar (*i.e.*, 96) to the right of the remainder. The new dividend is 496.

**Step IV :** Double the quotient and enter it with a blank in its right.

**Step V :** Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new quotient the product is less than or equal to the dividend. In this case we see that  $124 \times 4 = 496$ .

So the new digit in the quotient is 4. Get the remainder.

**Step VI :** Since the remainder is 0 and no bar left, therefore,  $\sqrt{4096} = 64$ .

**Cubes :**

- The cube of a number is that number raised to the power 3. If  $x$  is a number, then  $x^3 = x \times x \times x$ .
- A natural number  $n$  is a perfect cube, if  $n = m^3$  for some natural number  $m$ .
- The cube of an even natural number is even.
- The cube of an odd natural number is odd.
- The cube of a negative number is always negative.
- The sum of the cubes of first  $n$  natural numbers is equal to the square of their sum.  
i.e.,  $1^3 + 2^3 + 3^3 + \dots + n^3 = (1 + 2 + 3 + \dots + n)^2$
- Cubes of the numbers ending with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 end with digits 0, 1, 8, 1, 7, 5, 6, 3, 2, 9 respectively. Here, cubes of numbers ending with digits 0, 1, 4, 5, 6 and 9 end with same digits.
- Cubes of the number ending with digit 2 ends in 8 or cube of the number ending with digit 8 ends in 2.
- Cube of the number ending with digit 3 ends in 7 and cube of the number ending with digit 7 ends in 3.

**Cubes root :**

- The cube root of a number  $x$  is the number whose cube is  $x$ . It is denoted by  $\sqrt[3]{x}$ .
- For finding the cube root of a perfect cube, resolve it into prime factors; make triplets of similar factors and take the product of prime factors, choosing one out of every triplet.
- For any positive integer  $x$ , we have

$$\sqrt[3]{-x} = -\sqrt[3]{x}.$$

- For any integers  $a$  and  $b$ , we have :

$$(i) \sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

$$(ii) \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

**Ex.** Find the cube root of 17576 through estimation

**Sol.** The given number is 17576

**Step I :** Form groups of three starting from the rightmost-digit of 17576.

17 576. In this case one group i.e., 576 has three digits whereas 17 has only two digits.

**Step II :** Take 576

The digit 6 is at its one's place

We take the one's place of the required cube root as 6

**Step III :** Take the other group, i.e., - 17

Cube of 2 is 8 and cube of 3 is 27. 17 lies between 8 and 27.

The smaller number among 2 and 3 is 2.

The one's place of 2 is 2 itself. Take 2 at ten's place of the cube root of 17576.

Thus,  $\sqrt[3]{17576} = 26$

- Discount is a reduction given on marked price.

$$\text{Discount} = \text{Marked Price} - \text{Selling price.}$$

- Discount can be calculated when discount percentage is given.

$$\text{Discount} = \text{Discount \% of marked Price.}$$

- Additional expenses made after buying an article are included in the cost price and are known as overhead expenses.

$$\text{C.P.} = \text{Buying Price} + \text{Overhead expenses}$$

- Sales tax is charged on the sale of an item by the government and is added to the bill amount.

$$\text{Sales tax} = \text{Tax \% of bill amount}$$

- VAT (value added tax) is charged on the selling price of an article.

- **Percent** : The word percent is an abbreviation of the Latin phrase 'per centum' which means per hundred or hundredths.

$$\text{Sales Tax} = \frac{\text{C.P.} \times \text{Rate of S.T.}}{100}$$

- 

- M.P. = Marked Price,  
M.P. = S.P. + Discount,

$$\text{S.P.} = \text{Selling Price}$$

$$\text{Discount} = \text{M.P.} - \text{S.P.}$$

$$\text{Discount\%} = \frac{\text{Discount} \times 100}{\text{M.P.}}$$

- When profit % is given, then S.P. > C.P. and

$$(a) \quad \text{S.P.} = \left( \frac{100 + \text{Profit \%}}{100} \right) \times \text{C.P.}$$

$$(b) \quad \text{C.P.} = \frac{\text{S.P.} \times 100}{100 + \text{Profit \%}}$$

$$(c) \quad \text{Profit} = \frac{\text{Profit \%} \times \text{C.P.}}{100}$$

$$(d) \quad \text{Profit (gain)} = \text{S.P.} - \text{C.P.}$$

$$(e) \quad \text{Profit \%} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

- When loss % is given S.P. < C.P. and

$$(a) \quad \text{S.P.} = \frac{100 - \text{Loss\%}}{100} \times \text{C.P.}$$

$$(b) \quad \text{C.P.} = \frac{\text{S.P.} \times 100}{100 - \text{Loss\%}}$$

$$(c) \quad \text{Loss} = \frac{\text{Loss\%} \times \text{C.P.}}{100}$$

$$(d) \quad \text{Loss} = \text{C.P.} - \text{S.P.}$$

$$(e) \quad \text{Loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

- (f) Increase and Decrease Percent

$$\text{Increase \%} = \left( \frac{\text{Increase}}{\text{Original value}} \times 100 \right) \%$$

$$\text{Decrease \%} = \left( \frac{\text{Decrease}}{\text{Original value}} \times 100 \right) \%$$

- **Simple Interest (SI)** : When the interest is paid to the lender regularly every year or half year on the same interest, we call it as simple interest. In other words, interest is said to simple, if it is calculated on the original principle throughout the loan period.

and 
$$S.I. = \frac{P \times R \times T}{100}$$

Where, P = Principal, R = Rate of Interest, T = Time.

- **Compound Interest (CI)** : If the borrower and the lender agree to fix up a certain interval of time (say, a year or a half-year or a quarter of a year, etc.), so that the amount at the end of an interval becomes the principal for the next-interval, then the total interest over all the interval calculated in this way is called the compound interest.

Also, CI = Amount – Principal

- (a) When interest is compounded annually, then

$$\text{Amount } A = P \left(1 + \frac{R}{100}\right)^n \text{ and CI} = A - P = P \left\{ \left(1 + \frac{R}{100}\right)^n - 1 \right\}, \text{ where } P \text{ is Principal, } R \text{ is rate of interest and } n \text{ is time period}$$

- (b) When interest is compounded half yearly, then

$$\text{Amount } A = P \left(1 + \frac{R}{2 \times 100}\right)^{2n}, \text{ where } \begin{cases} \frac{R}{2} \text{ is half yearly rate and} \\ 2n = \text{number of half-years} \end{cases}$$

$$\text{and CI} = A - P = P \left\{ \left(1 + \frac{R}{2 \times 100}\right)^{2n} - 1 \right\}$$

- When  $R_1, R_2$  and  $R_3$  are different rates for first, second and third year, then

$$\text{Amount } A = P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right)$$

$$\text{and CI} = A - P = P \left\{ \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) - 1 \right\}$$

<p><b>Ex.</b> A sum of ₹ 20,000 is borrowed by a person for 2 years at an interest of 8% compounded annually. Find the compound interest (C.I) and the amount he has to pay at the end of 2 years.</p> <p><b>Sol. Step I :</b> Find the simple interest (S.I.) for one year. Let the principal of the first year be <math>P_1</math> Here, <math>P_1 = ₹ 20,000</math></p> <p><math>SI_1 = SI \text{ at } 8\% \text{ p.a for the } 1^{st} \text{ year} = \frac{20000 \times 8}{100} = ₹ 1600</math></p> <p><b>Step II :</b> Then Find the amount which will be paid or received. This becomes principal for the next year. Amount at the end of <math>1^{st}</math> year = <math>P_1 + SI_1</math></p>	<p style="text-align: right;">= ₹ 20000 + ₹ 1600 = ₹ 21600 = <math>P_2</math> (Principal of <math>2^{nd}</math> year)</p> <p><b>Step III :</b> Again find the interest on this sum for another year.</p> <p><math>SI_2 = SI \text{ at } 8\% \text{ p.a for } 2^{nd} \text{ year} = ₹ \frac{21600 \times 8}{100}</math> = ₹ 1728</p> <p><b>Step IV :</b> Find the amount which has to be paid or received at end of second year. Amount at the end of <math>2^{nd}</math> year = <math>P_2 + SI_2</math> = ₹ 21600 + ₹ 1728 = ₹ 23328</p> <p>Total interest given = ₹ 1600 + ₹ 1728 = ₹ 3328.</p>
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- A symbol which takes various numerical values is called a variable.
- A combination of constants and variables connected by the signs of fundamental operations of addition, subtraction, multiplication and division is called an algebraic expression.
- Various parts of an algebraic expression which are separated by the signs of '+' or '-' are called the terms of the expression.
- An algebraic expression is called a monomial, a binomial, a trinomial, a quadrinomial accordingly as it contains one term, two terms, three terms and four terms, respectively.
- While adding or subtracting polynomials, first look for like terms and then add or subtract these terms, then handle the unlike terms.
- Monomial multiplied by a monomial always gives a monomial.
- While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
- While multiplying a polynomial by a binomial (or trinomial), we multiply term by term. *i.e.*, every term of polynomial is multiplied by every term of in the binomial (or trinomial) and after that we combined the like terms.
- An identity is an equality, which is true for all values of the variable in the equality.
- The following are the useful identities and these identities are known as standard identities.
  - (i)  $(a + b)^2 = a^2 + 2ab + b^2$
  - (ii)  $(a - b)^2 = a^2 - 2ab + b^2$
  - (iii)  $(a + b)(a - b) = a^2 - b^2$
  - (iv)  $(x + a)(x + b) = x^2 + (a + b)x + ab$
  - (v)  $(x + a)(x - b) = x^2 + (a - b)x - ab$
  - (vi)  $(x - a)(x + b) = x^2 - (a - b)x - ab$
  - (vii)  $(x - a)(x - b) = x^2 - (a + b)x + ab$
  - (viii) If  $x$  is a variable and  $m, n$  are positive integers, then  
 $(x^m \times x^n) = x^{(m+n)}$ . Thus,  $x^2 \times x^4 = x^{(2+4)} = x^6$ ,
  - (ix) If  $x$  is a variable and  $m, n$  are positive integers such that  $m > n$ , then  
 $(x^m \div x^n) = x^{m-n}$ . Thus,  $x^9 \div x^4 = x^{9-4} = x^5$ .
  - (x) Product of two monomials = (Product of their coefficients)  $\times$  (Product of their variables)
  - (xi) Quotient of two monomials = (Quotient of their coefficients)  $\times$  (Quotient of their variables)

**Ex.** Evaluate  $(102)^2$  by using suitable identities.

**Sol. Step I :** Express the given number without-power on the sum or difference of two numbers.

Given number without power is 102. Since It is greater than 100. So it can be written on  $100 + 2$

$$\therefore (102)^2 = (100 + 2)^2$$

**Step II :** Compare the expression with  $(x + y)^2$  an company  $(100 + 2)^2$  with  $(x + y)^2$ , we get  $x = 100, y = 2$

**Step III :** Use the identify  $(x + y)^2 = x^2 + 2xy + y^2$  to expand it.

By using the identify

$$\begin{aligned} (x + y)^2 &= x^2 + 2xy + y^2 \\ (102)^2 &= (100 + 2)^2 \\ &= (100)^2 + 2(100)(2) + (2)^2 \end{aligned}$$

**Step IV :** Simplify the above expression.

$$\begin{aligned} (102)^2 &= 10000 + 400 + 4 \\ &= 10404. \end{aligned}$$

- Solids have fixed shape and occupy a space.
- A solid is made up of polygonal regions, which are called its faces.
- **Polyhedron** : A solid shape bounded by polygons is called polyhedron (platonic solid).

A polyhedron has some number of plane faces, edges and vertices, which satisfy the relationship :

$$F + V - E = 2,$$

where

'F' stands for number of faces.

'V' stands for number of vertices and

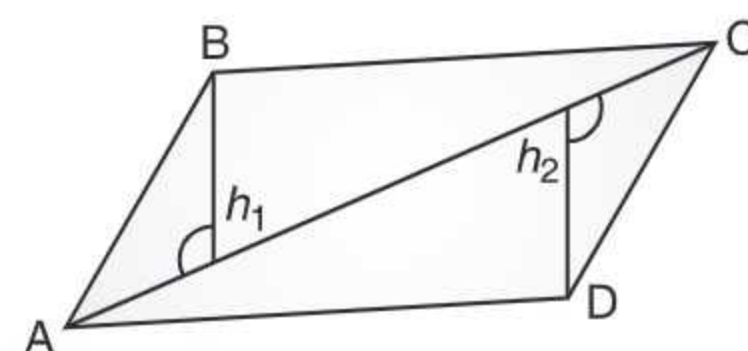
'E' stands for number of edges.

The relation  $F + V - E = 2$  is called Euler's formula.

- **Prism** : A prism is a solid, whose side faces are parallelograms and whose ends (or bases) are congruent parallel polygons.  
A prism has 2 triangular faces, 3 rectangular faces, 6 vertices and 9 edges.
- **Pyramid** : A pyramid is a polyhedron whose base is a polygon of any number of sides and whose other faces are triangles with a common vertex.  
A pyramid has 1 square face, 4 triangular faces, 5 vertices and 8 edges.
- **Tetrahedron** : A pyramid is called a triangular pyramid, if its base is a triangle. A triangular pyramid is also called a tetrahedron.  
Tetrahedron has 4 triangular faces, 4 vertices and 6 edges.
- **Hexahedron** : Cube is the only platonic solid whose every face is a square and cube is also known as hexahedron.  
A hexahedron has 6 square faces, 8 vertices and 12 edges.
- **Octahedron** : The platonic solid which has four equilateral triangles meeting at each vertex is known as octahedron.  
An octahedron has 8 triangular faces, 6 vertices and 12 edges.
- **Icosahedron** : The platonic solid in which 5 equilateral triangles meet a point to form a vertex, is known as an icosahedron.  
An icosahedron has 20 triangular faces, 12 vertices and 30 edges.
- **Dodecahedron** : The platonic solid whose every face is a pentagon is known as dodecahedron.  
A dodecahedron has 12 triangular faces, 20 vertices and 30 edges.

- The magnitude of a plane region is called its area.
- Length of boundary of a simple closed figure is known as perimeter.
- Perimeter of a rectangle =  $2(l + b)$ .
- Area of rectangle =  $l \times b$ .
- Perimeter of a square =  $4 \times \text{side}$ .
- Area of a square =  $(\text{side})^2$ .
- Area of a quadrilateral =  $\frac{1}{2} d(h_1 + h_2)$ .

where,  $d$  denotes the length of diagonal  $AC$ .



- Area of parallelogram = base  $\times$  height.
- Area of trapezium =  $\frac{1}{2} \times [\text{Sum of parallel sides}] \times \text{Height}$ .
- Area of an equilateral triangle =  $\frac{\sqrt{3}}{4} \times (\text{Side})^2$ .
- Area of a triangle =  $\frac{1}{2} \times \text{Base} \times \text{Height}$ .
- The perimeter of a circle is called its circumference.
- The ratio of circumference of a circle to its diameter is always constant and denoted by the Greek letter  $\pi$ . Thus,  $\frac{c}{d} = \pi$ . The value of  $\pi$  is 3.14 correct to two decimal places.
- The number  $\pi$  is not a rational number. It is often used as a rational approximation and its value is  $\frac{22}{7}$ .
- Circumference of a circle =  $2\pi \times \text{Radius} = 2\pi r$ .
- Area of a circle =  $\pi \times (\text{Radius})^2 = \pi r^2$ .
- Area of rhombus =  $\frac{1}{2} (\text{Product of diagonals}) = \frac{1}{2} \times d_1 \times d_2$ .
- Surface area of
  - (a) a cuboid =  $2[lb + bh + hl]$
  - (b) a cube =  $6l^2$
  - (c) a cylinder =  $2\pi r(h + r)$
  - (d) Lateral surface area of cuboid =  $[2(l + b) \times h]$  square unit
  - (e) Diagonal of cuboid =  $\sqrt{l^2 + b^2 + h^2}$  units
  - (f) Lateral surface area of the cube =  $[4a^2]$  square unit
  - (g) Lateral (curved) surface area of a cylinder =  $2\pi rh$
- Volume of
  - (a) cuboid =  $l \times b \times h$
  - (b) cube =  $l^3$
  - (c) cylinder =  $\pi r^2 h$
  - (d) Diagonal at the cube =  $(\sqrt{3}a)$  units.
- (a)  $1 \text{ m}^2 = 100 \text{ dm}^2 = 10000 \text{ cm}^2$
- (b)  $1 \text{ cm}^2 = 100 \text{ mm}^2$
- (c)  $1 \text{ hectare} = 10000 \text{ m}^2 = \frac{1}{100} \text{ km}^2$
- (d)  $1 \text{ cm}^3 = 1 \text{ ml}$
- (e)  $1 \text{ L} = 1000 \text{ cm}^3$
- (f)  $1 \text{ m}^3 = 10,00,000 \text{ cm}^3 = 1,000 \text{ L}$

# CHAPTER 12

# EXPONENTS AND POWERS

➤ The numbers with negative exponents also obey the following laws :

(a)  $x^m \times x^n = x^{m+n}$

(b)  $x^m \div x^n = x^{m-n}$

(c)  $x^m \times b^m = (xb)^m$

(d)  $x^0 = 1$

(e)  $\frac{x^m}{y^m} = \left(\frac{x}{y}\right)^m$

(f)  $\left(\frac{x}{y}\right)^n = \left(\frac{y}{x}\right)^{-n}$

(g)  $\frac{1}{x} = x^{-1}$

(h)  $x^p \times x^q \times x^r \times x^s = x^{p+q+r+s}$

(i)  $[(x^m)^n = x^{mn}]$

➤ A number is said to be in the standard form, if it is expressed as the product of a number between 1 and 10 and the integral power of 10.

e.g.,  $149,600,000,000 = 1.496 \times 10^{11}$

➤ Very small numbers can be expressed in standard form using negative exponents.

e.g.,  $0.0016 = \frac{16}{10000} = 1.6 \times 10^{-3}$

**Ex.** Express  $\left(\left(\frac{-3}{2}\right)^{-2}\right)^{-3}$  as a power of a rational number with negative exponent.

**Sol. Step I :** Using laws of exponents,  $(a^m)^n = (a)^{m \times n}$  and  $a^{-m} = \frac{1}{a^m}$  [ $\because$  a is non-zero integer]

$$\therefore \left(\left(\frac{-3}{2}\right)^{-2}\right)^{-3} = \left(\frac{-3}{2}\right)^{-2 \times (-3)} = \left(\frac{-3}{2}\right)^6 = \left(\frac{-2}{3}\right)^{-6}$$

**Step II :** Using law of exponents,

$$a^m \div a^n = a^{m-n}$$

and  $a^m \times a^n = a^{m+n}$  [ $\because$  a is non-zero integer]

$$\begin{aligned} \therefore (2^5 \div 2^8) \times 2^{-7} &= 2^{5-8} \times 2^{-7} \\ &= 2^{-3} \times 2^{-7} \\ &= 2^{-3} \times 2^{-7} = 2^{-3-7} = 2^{-10} \end{aligned}$$

# CHAPTER 13

## DIRECT AND INVERSE PROPORTIONS

➤ **Direct proportion :**

(a) Two quantities  $x$  and  $y$  are said to be in direct proportion if whenever the value of  $x$  increases (or decreases), then the value of  $y$  increases (or decreases) in such a way that the ratio  $\frac{x}{y}$  remains constant.

(b) When  $x$  and  $y$  are in direct proportions, we have :

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3}, \text{ etc.}$$

➤ **Inverse proportion :**

(a) Two quantities  $x$  and  $y$  are said to be in inverse proportion, if whenever the value of  $x$  increases (or decreases), then the value of  $y$  decreases (or increases) in such a way that  $xy$  remains constant.

(b) When  $x$  and  $y$  are in inverse proportion, then

$$x_1 \times y_1 = x_2 \times y_2 = x_3 \times y_3, \text{ and so on.}$$

<p><b>Ex.</b> An electric pole, 14 meters high, casts a shadow of 10 meters. Find the height of a tree that casts a shadow of 15 meter under similar conditions.</p> <p><b>Sol.</b> Let the height of tree be <math>x</math> meters.</p> <p><b>Step I :</b> First form a table</p> <table border="1" style="width: 100%; border-collapse: collapse; margin: 5px 0;"> <tr> <td style="padding: 2px;">Height of object (in meters)</td> <td style="padding: 2px; text-align: center;">14</td> <td style="padding: 2px; text-align: center;"><math>x</math></td> </tr> <tr> <td style="padding: 2px;">Length of shadow (in meters)</td> <td style="padding: 2px; text-align: center;">10</td> <td style="padding: 2px; text-align: center;">15</td> </tr> </table> <p><b>Step II :</b> Use <math>\frac{x_1}{y_1} = \frac{x_2}{y_2}</math></p> <p>We have <math>\frac{14}{10} = \frac{x}{15}</math></p>	Height of object (in meters)	14	$x$	Length of shadow (in meters)	10	15	<p><b>Step III :</b> Solve is for finding <math>x</math>.</p> <p>i.e., <math>x = \frac{14 \times 15}{10}</math></p> $= \frac{14 \times 3}{2}$ $= 7 \times 3$ $x = 21$ <p>Thus height of the tree is 21 meters.</p>
Height of object (in meters)	14	$x$					
Length of shadow (in meters)	10	15					

# CHAPTER 14

# FACTORISATION

- When an expression is the product of two or more expressions, then each of the expressions is called a factor of the given expression.
- The process of writing a given expression as the product of two or more factors is called factorization.
- The greatest common factor of two or more monomials is the product of the greatest common factors of the numerical coefficients and the common letters with smallest powers.
- When a common monomial factor occurs in each term of an algebraic expression, then it can be expressed as a product of the greatest common factor of its terms and quotient of the given expression by the greatest common factor of its terms.
- When a binomial is a common factor, we write the given expression as the product of this binomial and the quotient of the given expression by this binomial.
- If the given expression is the difference of two squares, then to factorize it, we use the formula :  
 $(a^2 - b^2) = (a + b)(a - b)$
- If the given expression is a complete square, we use one of the following formulae to factorize it :  
 (i)  $a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$   
 (ii)  $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$
- For factorisation of the form  $(x^2 + px + q)$ , we find two numbers  $a$  and  $b$  such that  $(a + b) = p$  and  $ab = q$ , then  
 $x^2 + px + q = x^2 + (a + b)x + ab = (x + a)(x + b)$ .
- In case of division of a polynomial by a monomial, we may carry out the division either by dividing each term of the polynomial by the monomial or by the common factor method.
- In case of division of polynomial by a polynomial, we cannot proceed by dividing each term in the dividend polynomial by the division polynomial. Instead, we factorise both the polynomial and cancel their common factors.
- In the case of division of algebraic expression, we have  $\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$

**Ex.** Factorise  $6xy - 4y + 6 - 9x$

**Sol. Step I :** Check if there is a common factor among all terms. There is none.

**Step II :** Think of grouping. Notice that first two terms have a common factor  $2y$ .

$$6xy - 4y = 2y(3x - 2) \quad \dots(1)$$

For Last two terms, if we change the order the factor  $(3x - 2)$  will come out.

*i.e.,*  $-9x + 6 = -3(3x - 2) \quad \dots(2)$

**Step III :** Putting (1) and (2) together

$$\begin{aligned} 6xy - 4y + 6 - 9x &= 6xy - 4y - 9x + 6 \\ &= 2y(3x - 2) - 3(3x - 2) \\ &= (3x - 2)(2y - 3) \end{aligned}$$

The factors of  $(6xy - 4y + 6 - 9x)$  are  $(3x - 2)$  and  $(2y - 3)$

- The  $x$ -coordinate of a point is its distance from  $y$ -axis.
- The  $y$ -coordinate of a point is its distance from  $x$ -axis.
- The coordinates of the origin are  $(0, 0)$ .
- The  $x$ -coordinate of every point on  $y$ -axis is zero.
- The  $y$ -coordinate of every point on  $x$ -axis is zero.
- A bar graph is used to show comparison among categories.
- A pie graph is used to compare parts of a whole.
- A histogram is a bar graph that shows data in intervals.
- A line graph displays data that changes continuously over periods of time.
- A line graph which is a whole unbroken line is called a linear graph.
- Fixing a point on the graph sheet we need,  $x$ -coordinate and  $y$ -coordinate.
- The relation between dependent variable and independent variable is shown through a graph.

**Ex.** The following tables gives the quantity of petrol and its cost.

No. of litres of Petrol	10	15	20	25
Cost of petrol in ₹	500	750	1000	1250

Plot a graph to show the data.

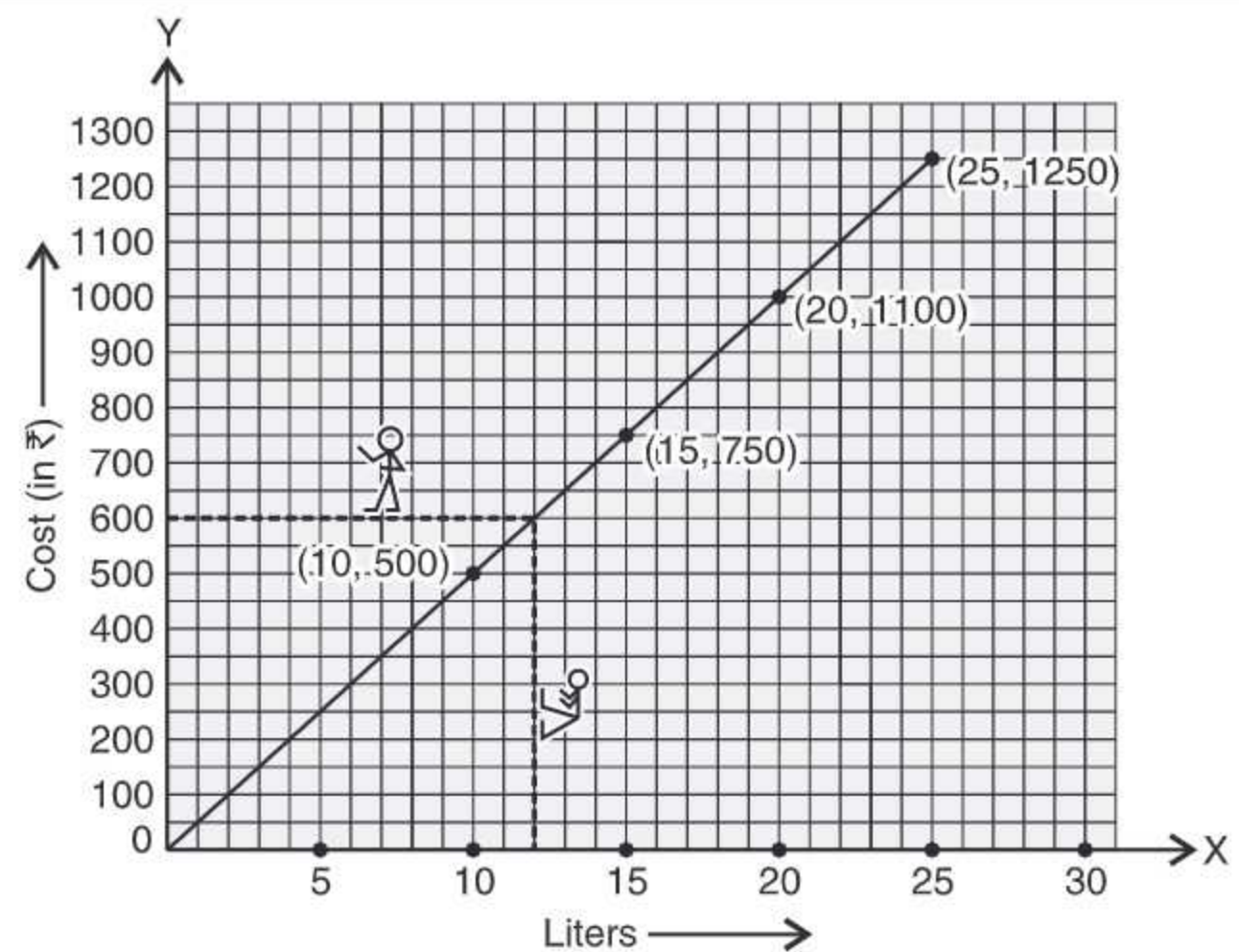
**Sol.** **Step I :** Let us take a suitable scale on both the axis.

**Step II :** Mark number of litres along the horizontal axis.

**Step III :** Mark cost of petrol along the verticals axis

**Step IV :** Plot the points :  $(10, 500)$ ,  $(15, 750)$ ,  $(20, 1000)$ ,  $(25, 1250)$ .

**Step V :** Join the points.



We find that the graph is a line. (It is a linear graph.)

# CHAPTER 16

## PLAYING WITH NUMBERS

- Any two digit number can be written as  $ab$ , where  $a$  and  $b$  are its tens and ones digit respectively.
- (a) A 2-digit number is of the form  $10a + b$ , where  $a$  is any whole number from 1 to 9 and  $b$  is any whole number from 0 to 9.
- (b) A 3-digit number is of the form  $100a + 10b + c$ , where  $a$  is any whole number from 1 to 9,  $b$  is any whole number from 0 to 9 and  $c$  is any whole number from 0 to 9.
- **Test of divisibility :**
  - (a) A number is divisible by 2 only when its unit digit is even.
  - (b) A number is divisible by 3 only when the sum of its digits is divisible by 3.
  - (c) A number is divisible by 4 if the number formed by its last two digits is divisible by 4.
  - (d) A number is divisible by 5 only when its unit digit is 0 or 5.
  - (e) A number is divisible by 6, if it is divisible by both 2 and 3.
  - (f) A number is divisible by 8, if it is divisible by both 2 and 4.
  - (g) A number is divisible by 9 only when the sum of its digits is divisible by 9.
  - (h) A number is divisible by 10, if its unit digit is zero.
- A number is even or odd according to its ones digit.

**Ex.** Check the divisibility of 21436587 by 9.

**Sol. Step I :** A number  $N$  is divisible by 9 if the sum of its digit is divisible by 9.

Hence sum of digits of 21436587

$$= 2 + 1 + 4 + 3 + 6 + 5 + 8 + 7 = 36$$

$$36 \div 9 = 4$$

**Step II :** Divide the given number 9.

*i.e.* 
$$\frac{21436587}{9} = 2381843$$